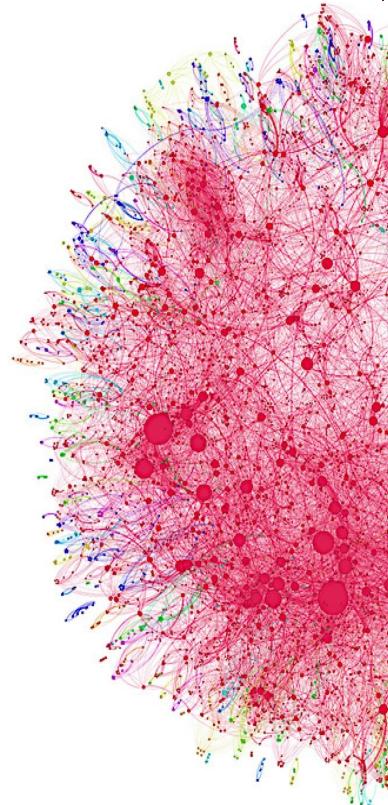


# sGradd: Towards RELIABLE Stream...ing Graph Analytics

Aida Sheshbolouki

Supervisor: Prof. M. Tamer Ozsu

David R. Cheriton School of Computer Science, University of Waterloo





Graphs are everywhere!



Finance

Transportation

Social Media Use

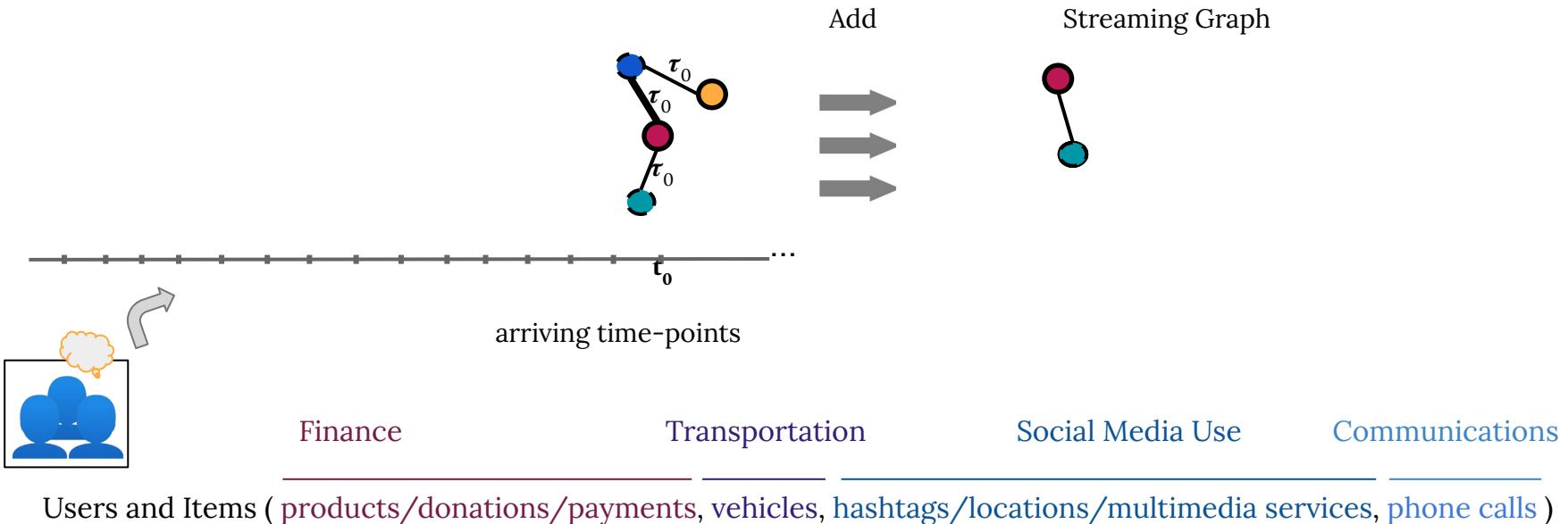
Communications

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Users and Items ( products/donations/payments, vehicles, hashtags/locations/multimedia services, phone calls )

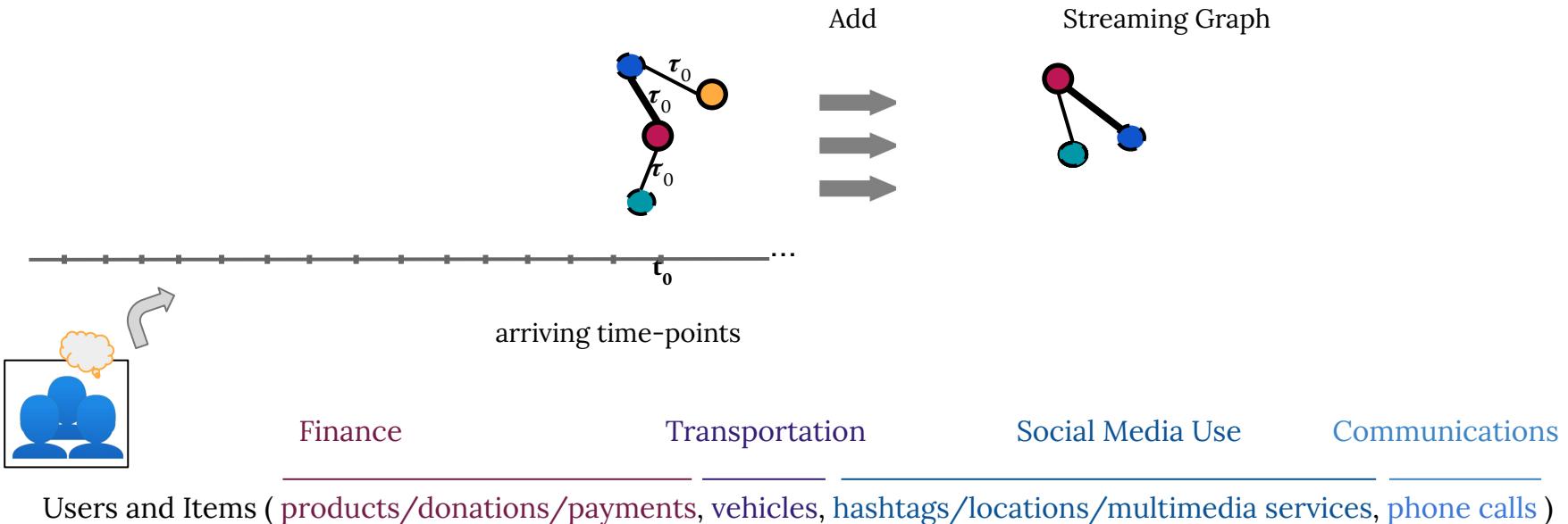
## An unbounded stream of linked data records

$(\textcolor{violet}{\circlearrowleft}, \textcolor{purple}{\circ}, \textcolor{black}{\equiv}, \textcolor{teal}{\tau})$   
user item weight generation timestamp



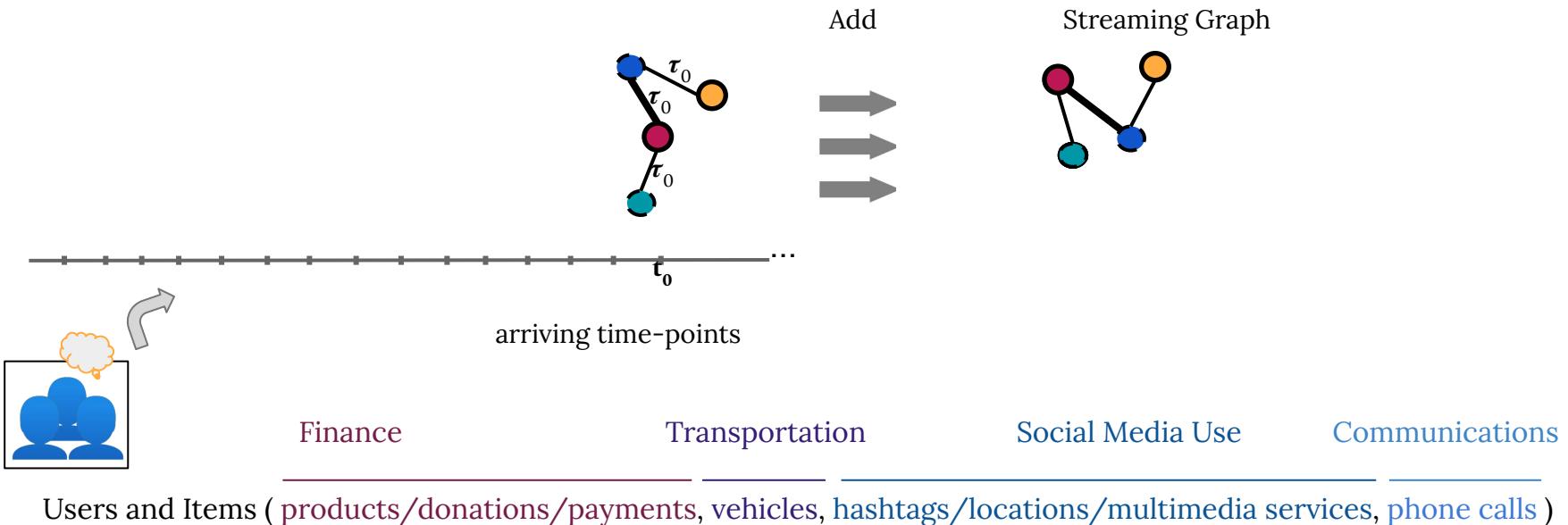
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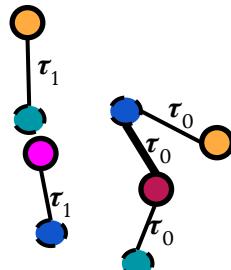
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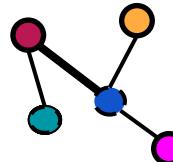
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Add

Streaming Graph



arriving time-points

Finance

Transportation

Social Media Use

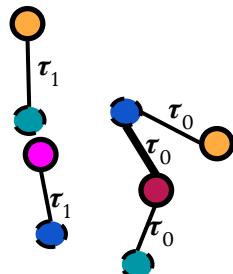
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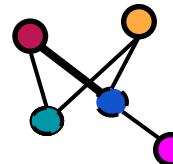
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Streaming Graph



$t_1$   $t_0$  ...  
arriving time-points

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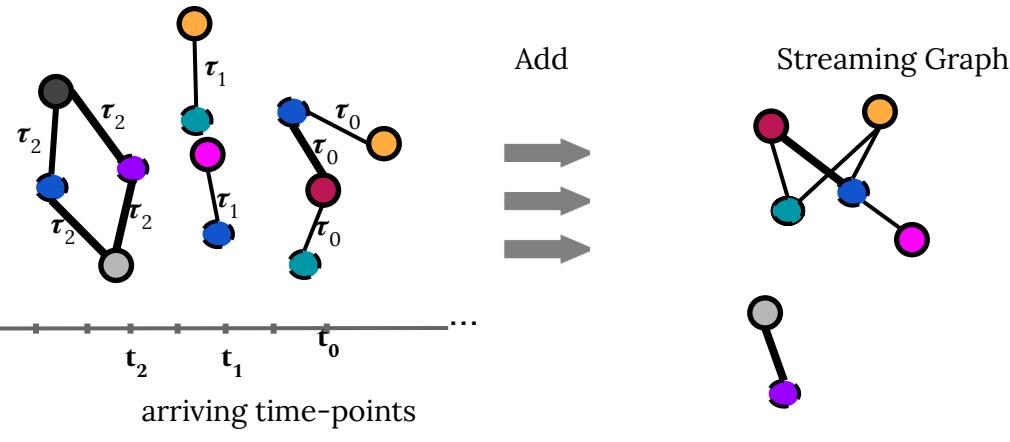
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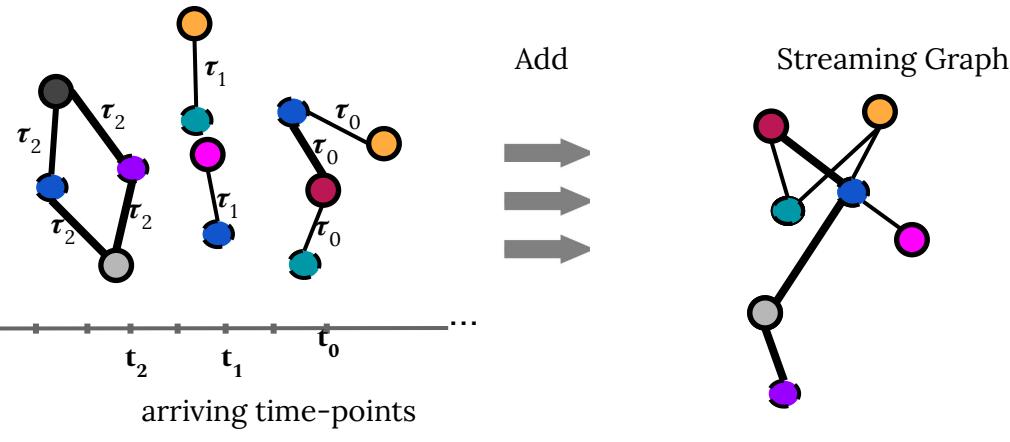
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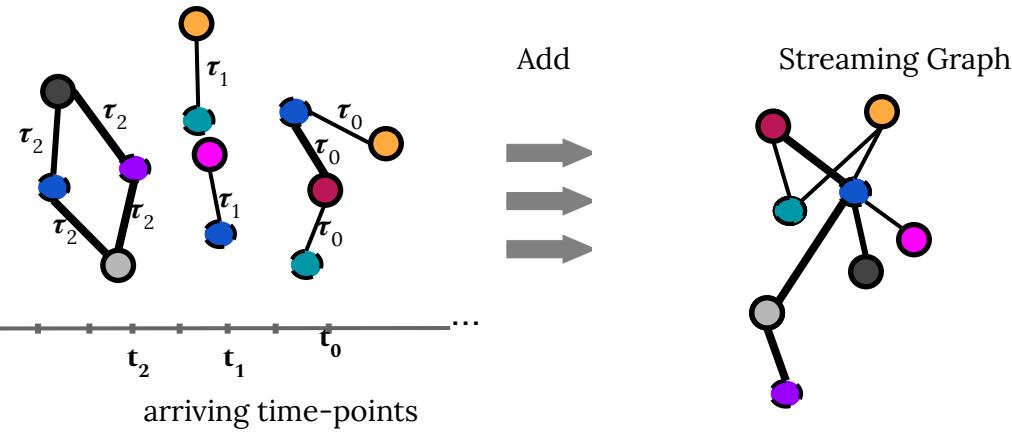
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Users and Items ( products/donations/payments, vehicles, hashtags/locations/multimedia services, phone calls )

## An unbounded stream of linked data records

(, , , )  
user item weight generation timestamp



## Finance

## Transportation

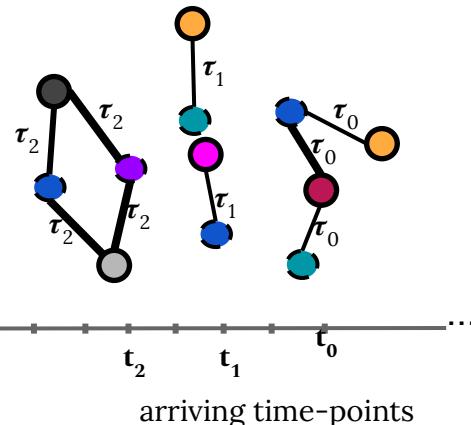
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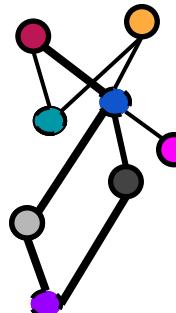
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Add

Streaming Graph



Finance

Transportation

Social Media Use

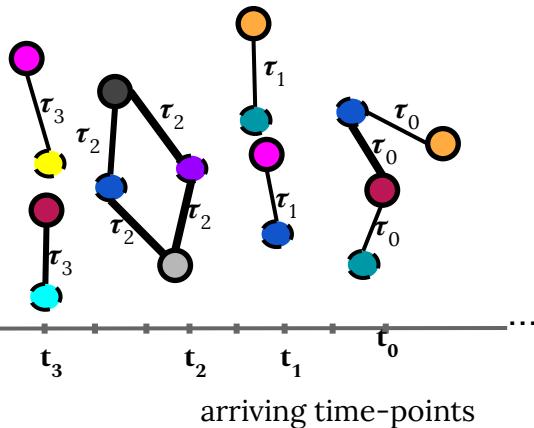
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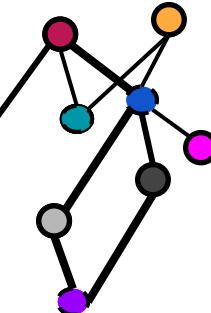
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Add



Streaming Graph



Finance

Transportation

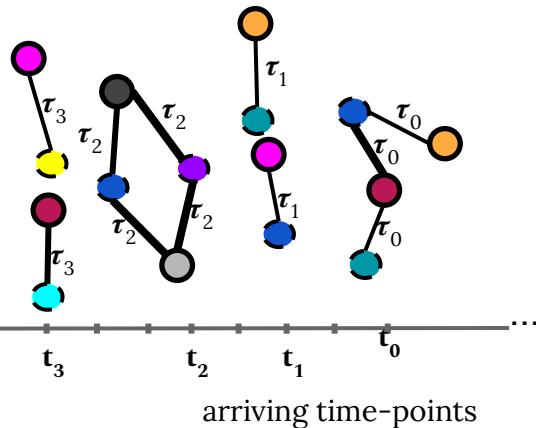
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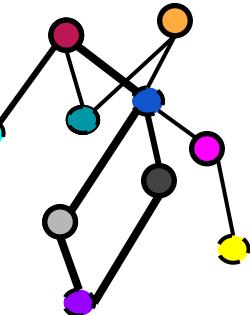
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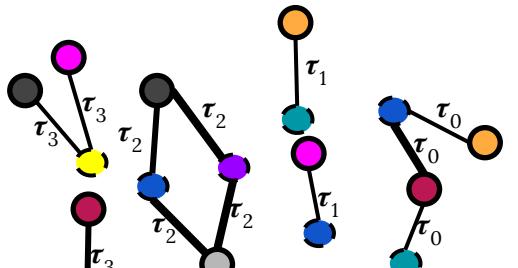
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(, , ,  $\tau$ )

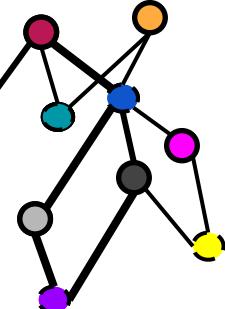
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Streaming Graph



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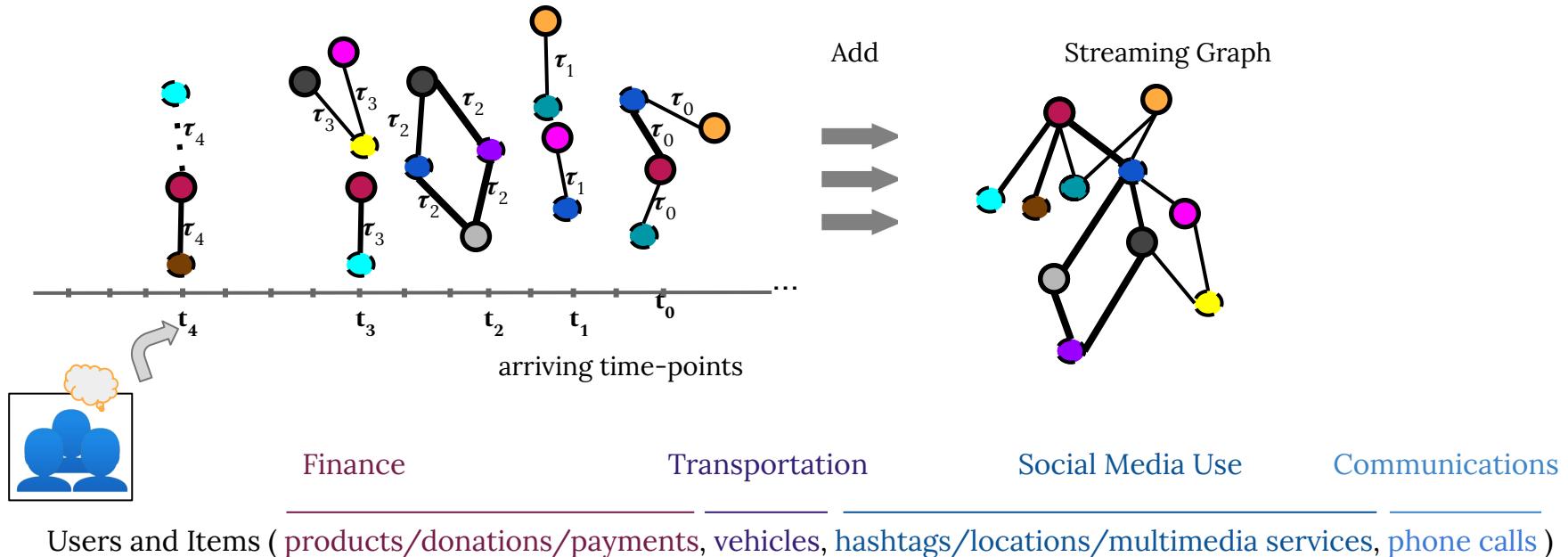
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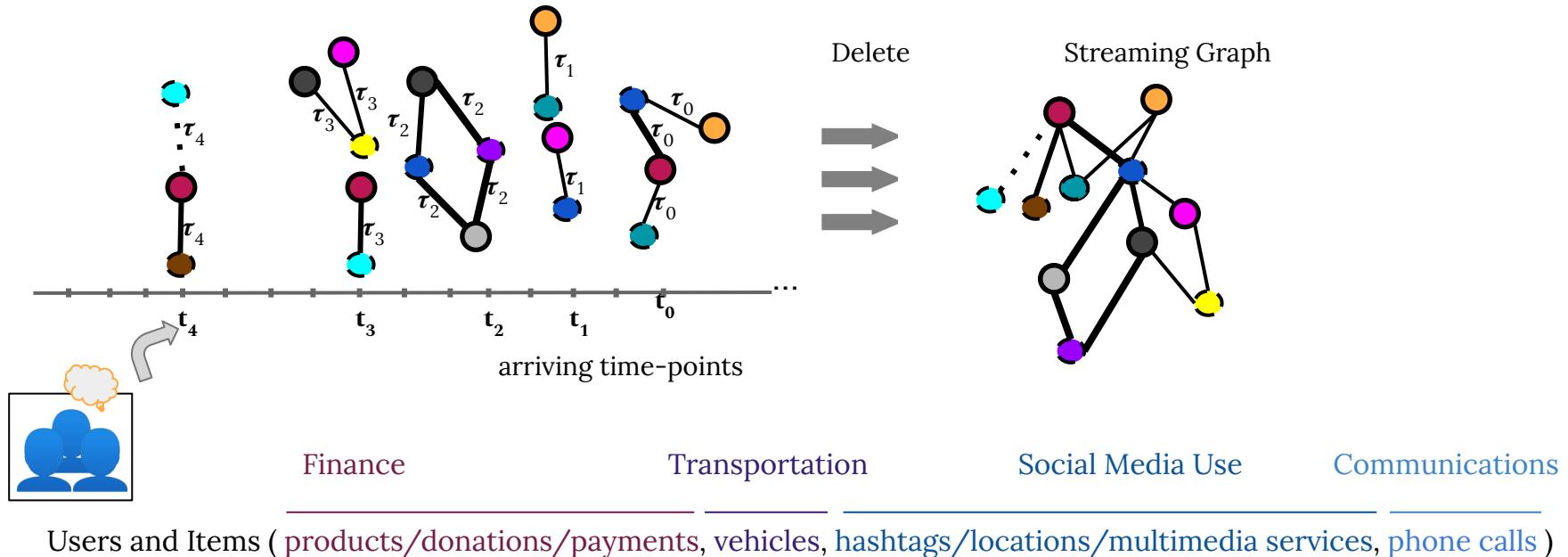
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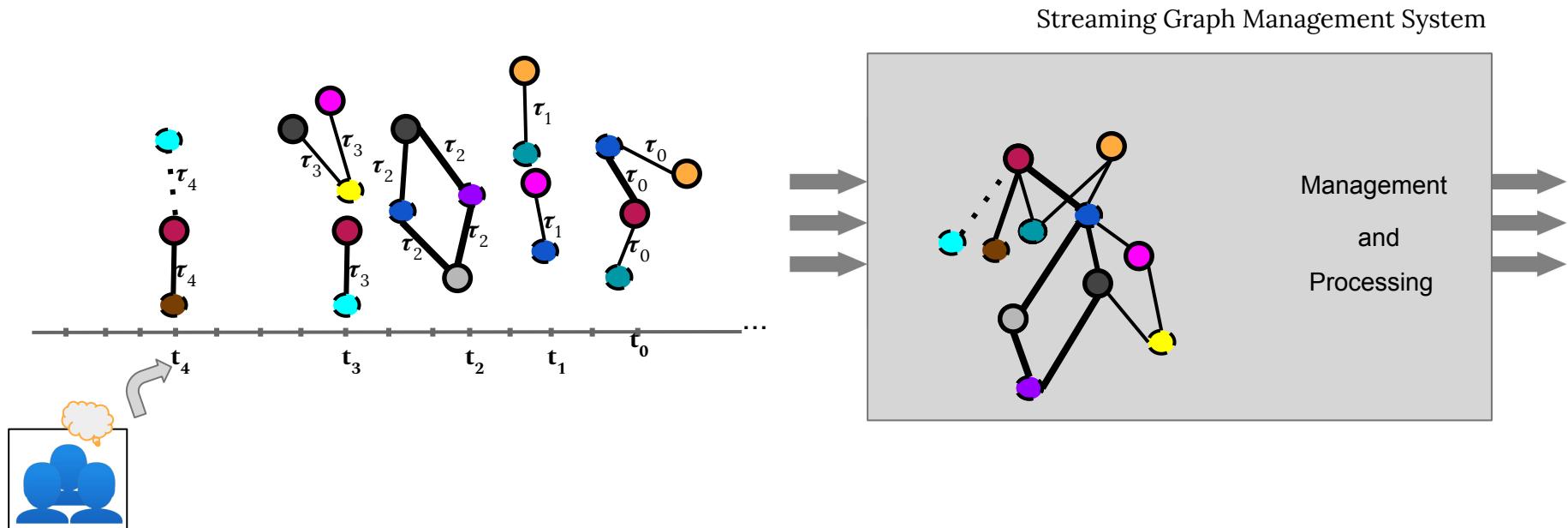


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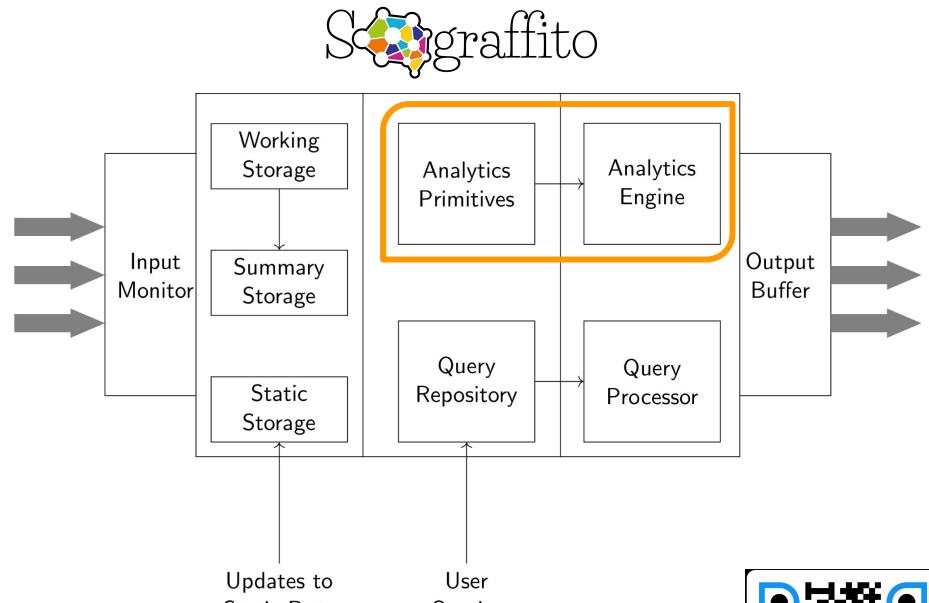
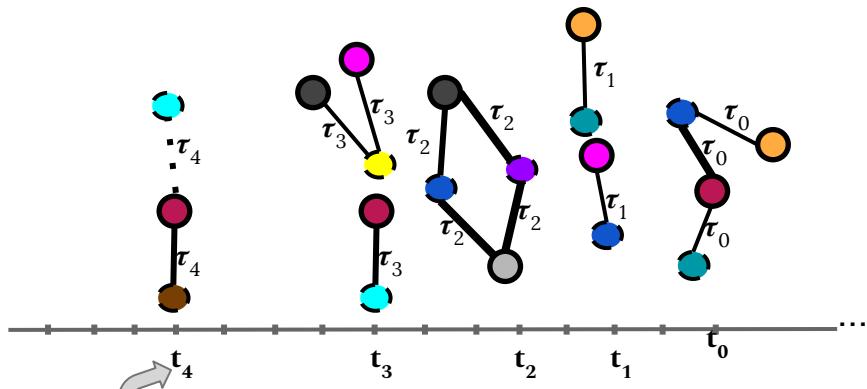


## An unbounded stream of linked data records



An unbounded sequence of partially ordered **Streaming Graph** Records =  $\langle r^1, r^2, \dots \rangle$

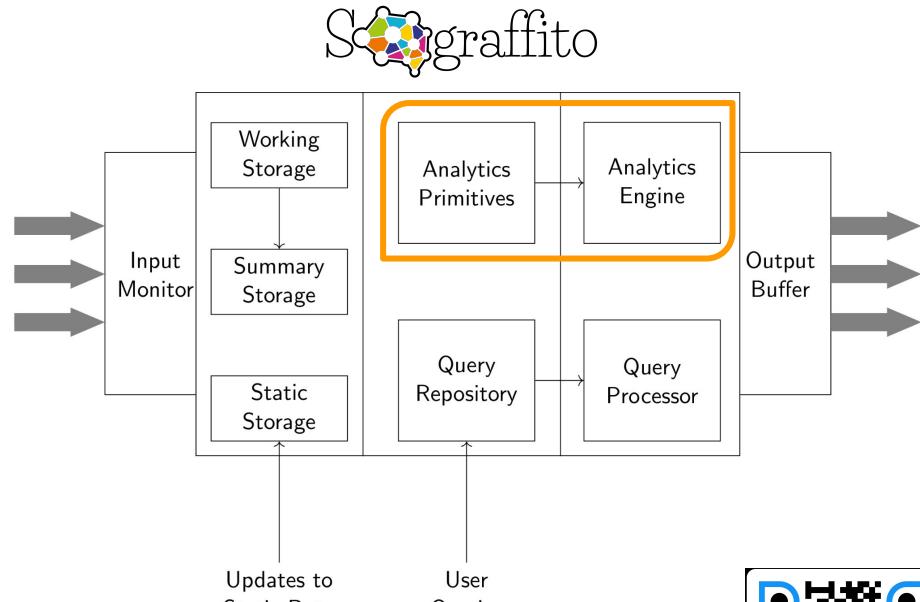
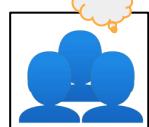
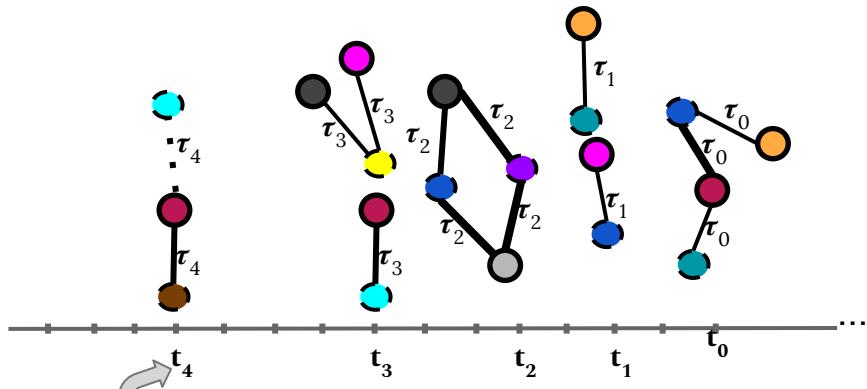
$$r^m = \langle v_i^m, v_j^m, w_{ij}^m, \tau^m \rangle, t^m \quad t^m \leq t^n \text{ for } m < n$$



- Dynamic and high volume/velocity
- Reliable output

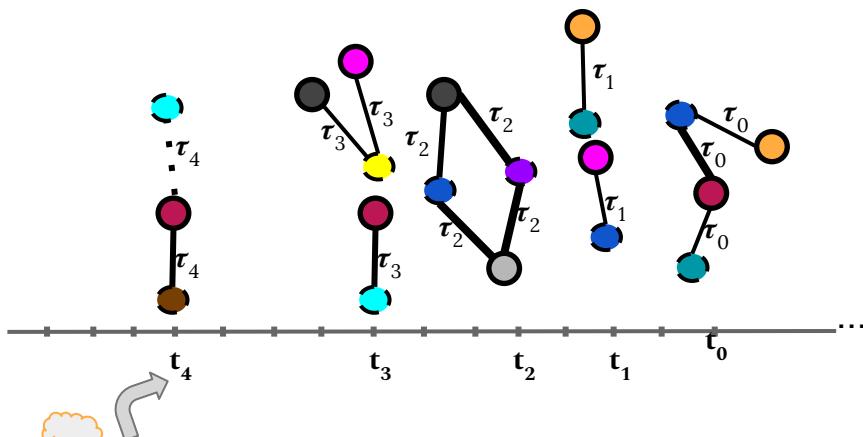
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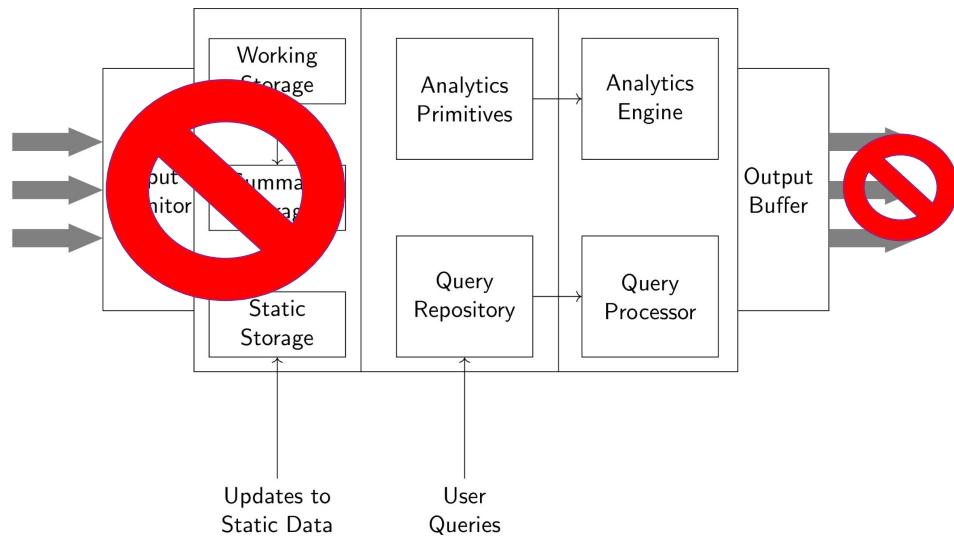


Unreliable Input + Mismanagement = Unreliable Output

→ New,  
Temporal,  
Incomplete, or  
Adversely manipulated



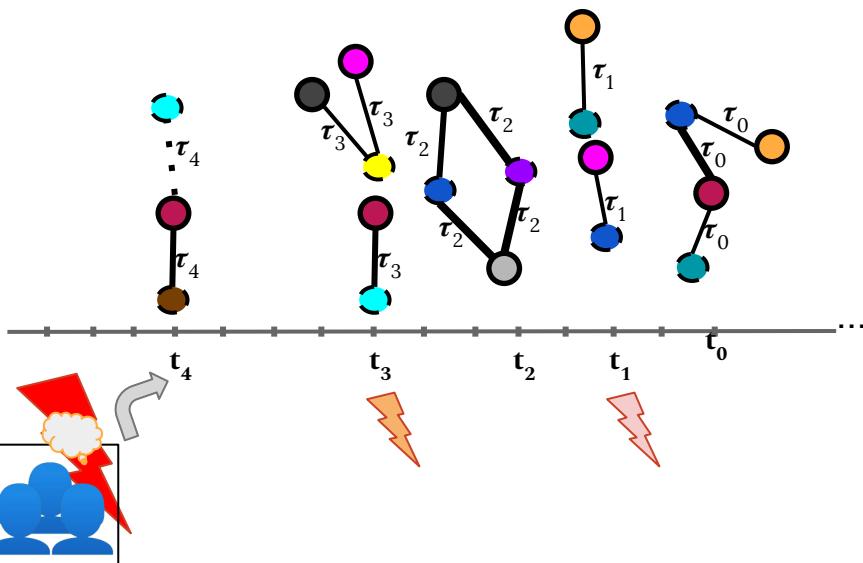
graffito



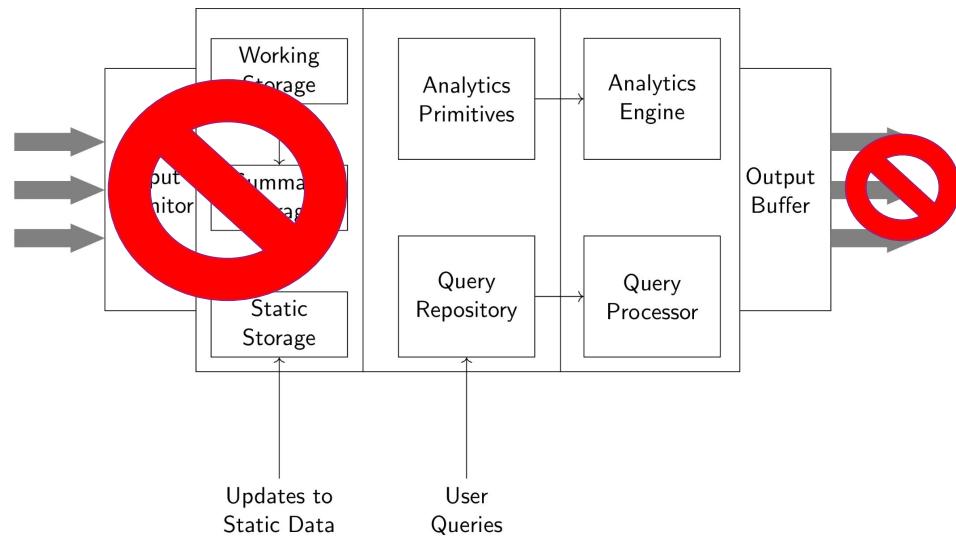
A main cause: **Concept Drift (CD)** occurs when a change in a hidden context induces changes in a target concept.

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Scraffito



A main cause: **Concept Drift (CD)** occurs when a change in a hidden context induces changes in a target concept.

Unreliable Input + **Drift Management** = Reliable Input

New,

Temporal,

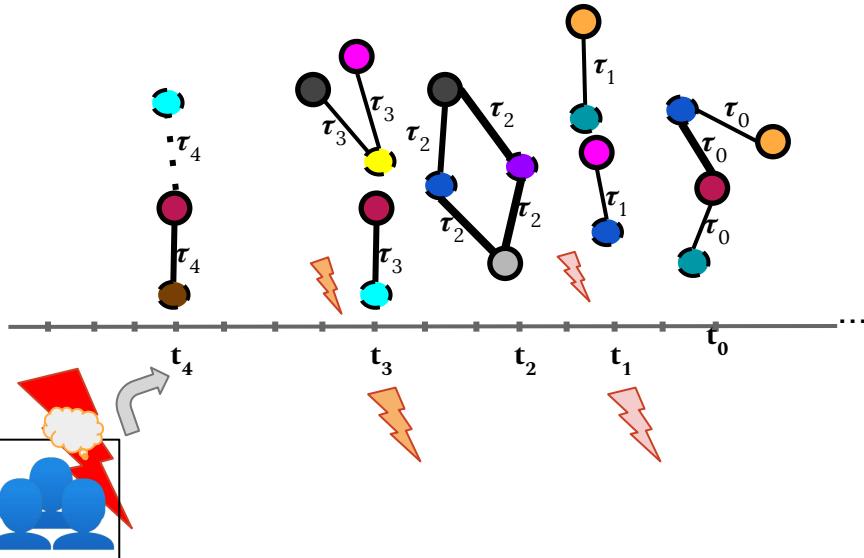
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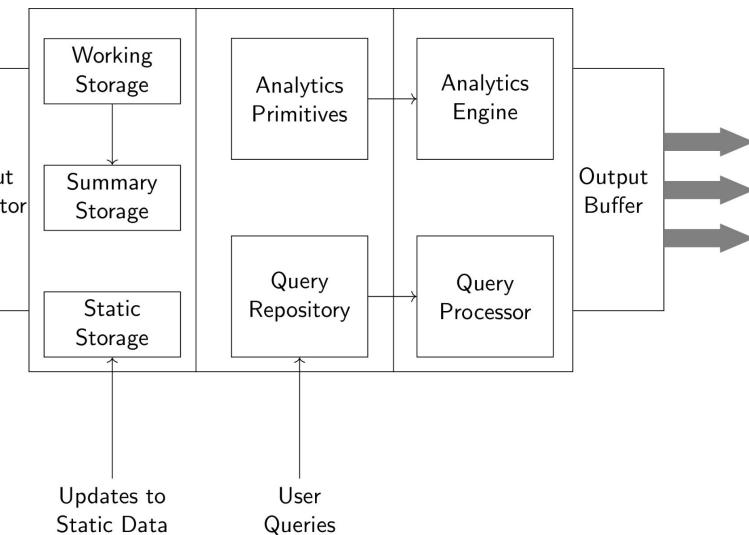
**CD Understanding**

**CD Detection**

**CD Adaptation**



Sograffito



How to detect concept drift in streaming graphs?

# Existing Works on CD detection

## **Data**

Sequence of independently generated data records

Sequence of attributed/labelled graph snapshots

## **Methods**

Integrated drift detection and adaptation within supervised learners

Parameterised and static techniques

# CD in Streaming Graphs

**Structural pattern**  $\beta$  refers to a quantified pattern of inter-connectivities of characteristic sub-structures (i.e. motifs) in a graph snapshot.

$$\beta(G_{W,t}) : G_{W,t} \rightarrow R$$

**Transient concept** refers to a non-stable structural pattern in transient data records.

$$\beta(G_{W,t}) \mid \exists (W_1, t_1), (W_2, t_2) : \beta(G_{W_1, t_1}) \neq \beta(G_{W_2, t_2})$$

**Concept drift in a streaming graph** is the event of a change in a transient concept over successive graph snapshots. Given a certain pattern  $\beta$ , concept drift can be detected when observing at least two successive windows  $W_1$  and  $W_2$  corresponding to sequential time points  $t_1$  and  $t_2$ , where  $t_2 - t_1 \geq 1$  and  $\beta(G_{W_1, t_1}) \neq \beta(G_{W_2, t_2})$ .

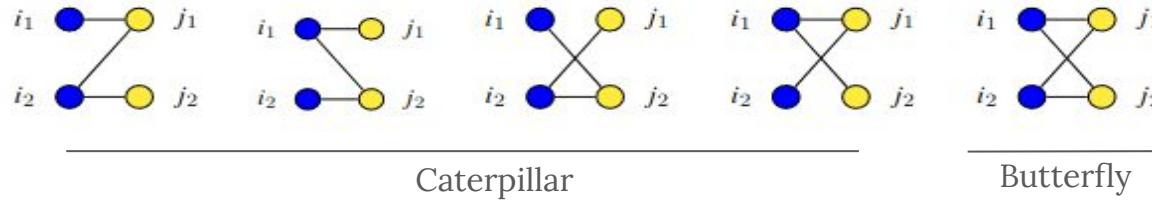
Detect changes and triggers updates

=

Concept Drift Detection

Goal

Focus on  
butterflies!



A **butterfly** forms by adding a link to a caterpillar.

# Mining Butterflies in Streaming Graphs

Aida Sheshbolouki (2023). UWspace



UNIVERSITY OF  
WATERLOO

FACULTY OF  
MATHEMATICS



Detect changes and triggers updates  
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**Concept Drift Detection**

Focus on  
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Goal

## Stream Mining

Discover and formulate  
the emergence patterns of  
butterflies in streaming graphs



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**sGrapp**: A framework for

Streaming Graph Butterfly **Approximation**

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Explainable Modelling

**sGrow**: A framework for  
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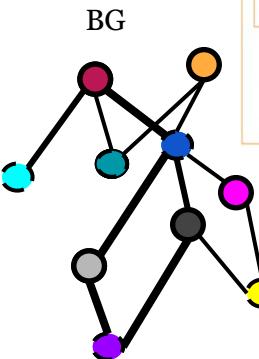
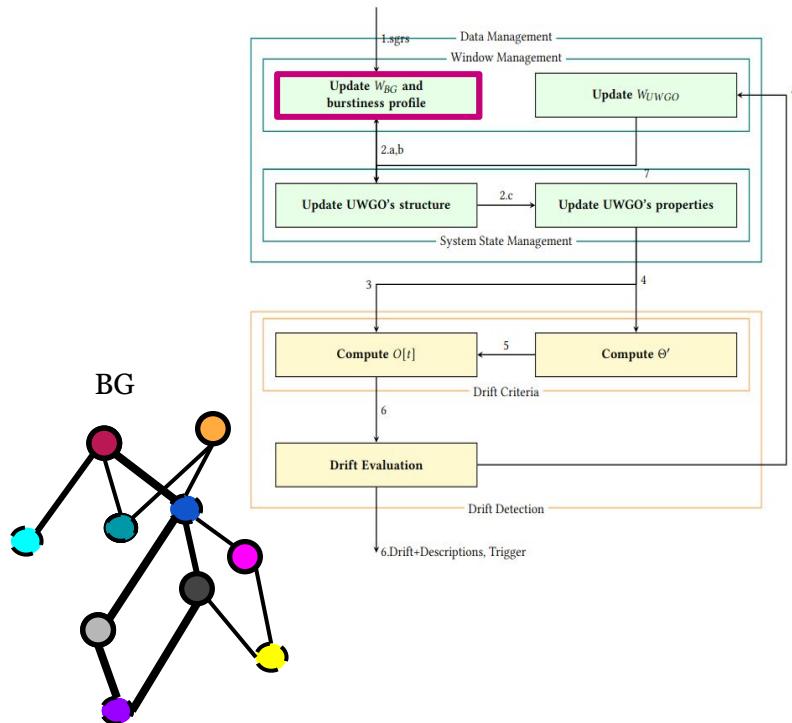
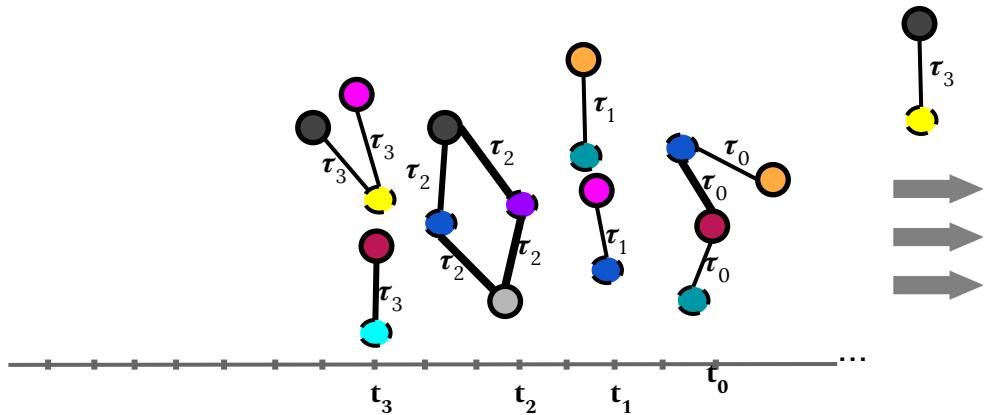
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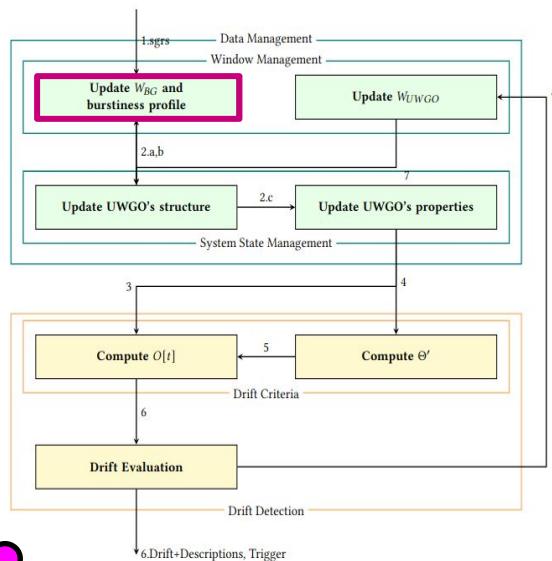
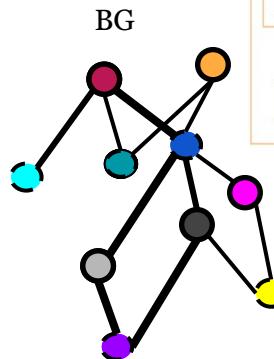
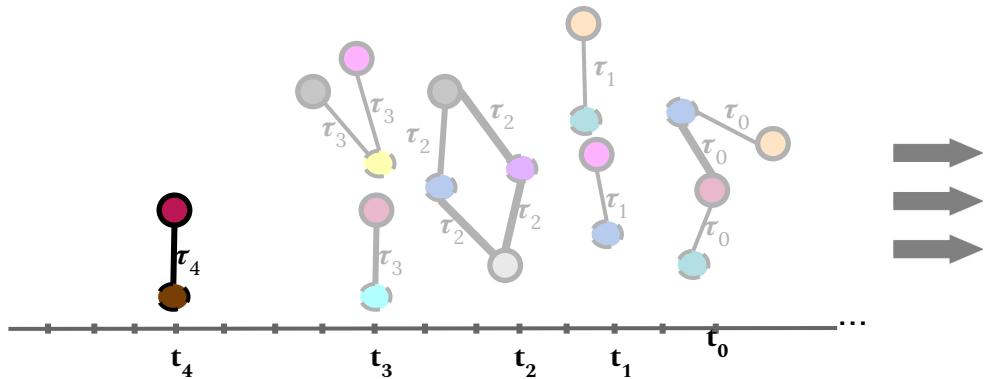
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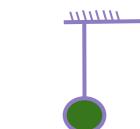
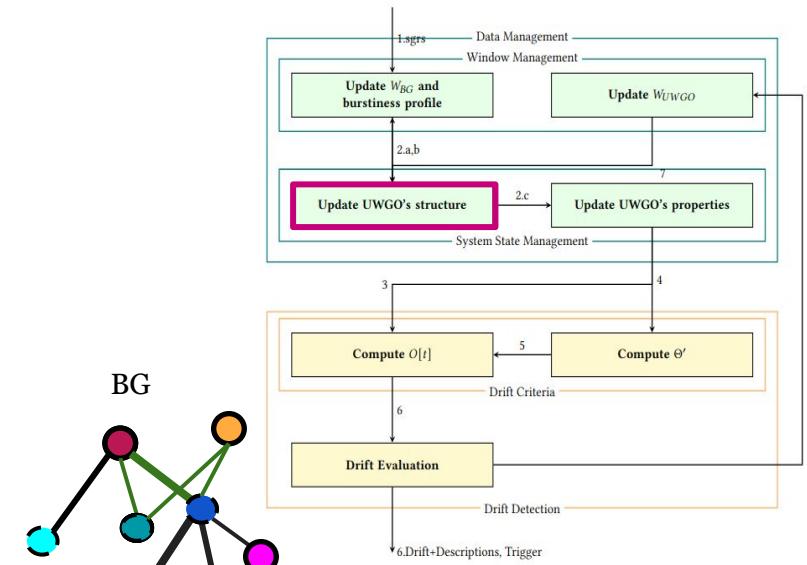
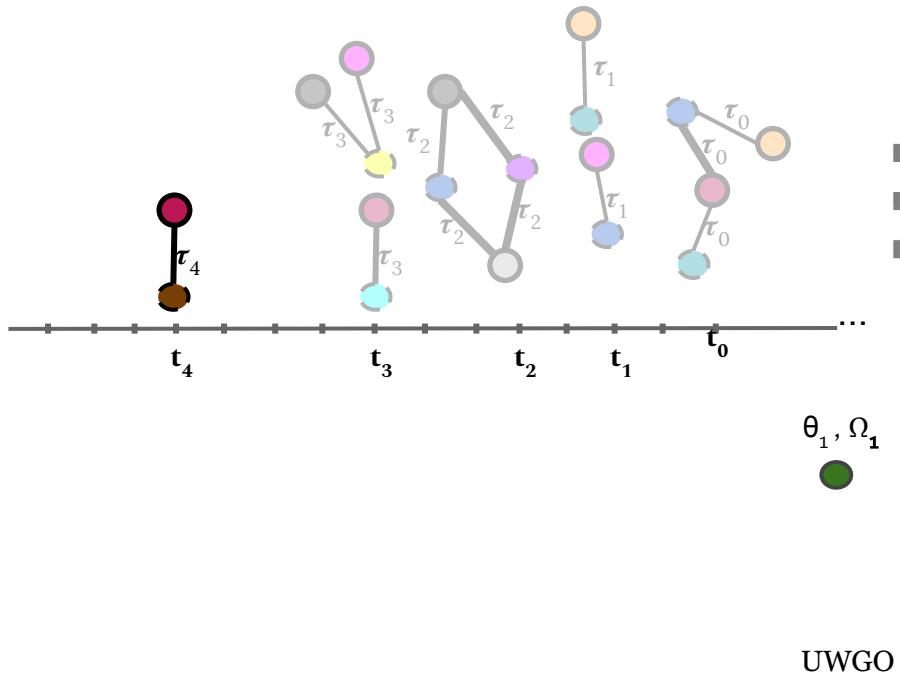
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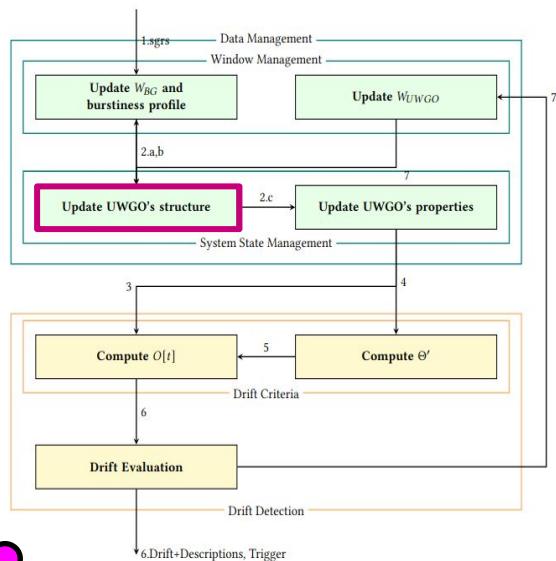
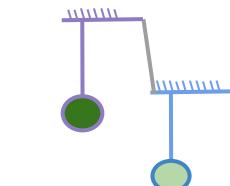
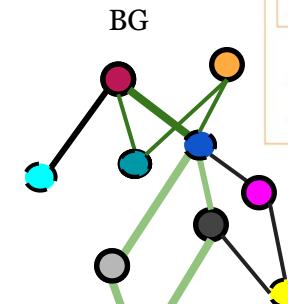
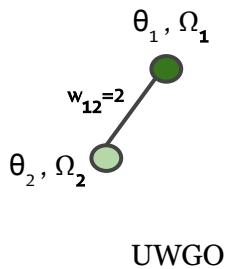
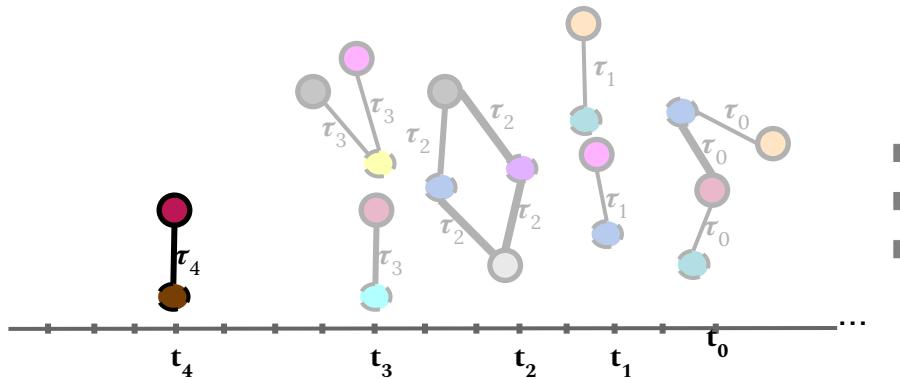


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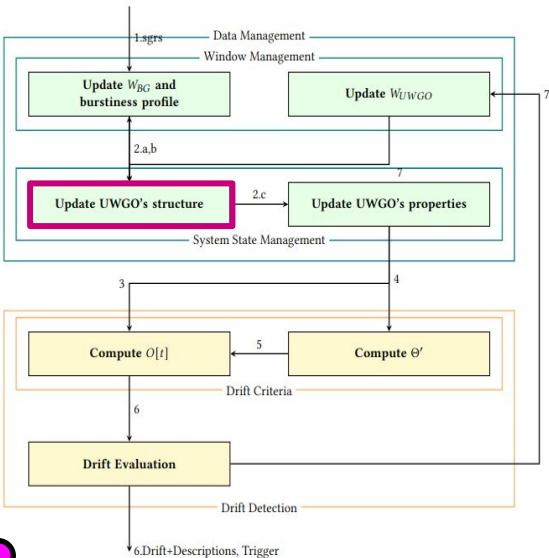
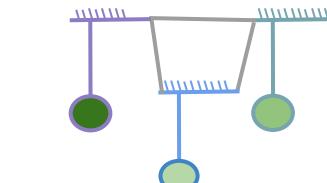
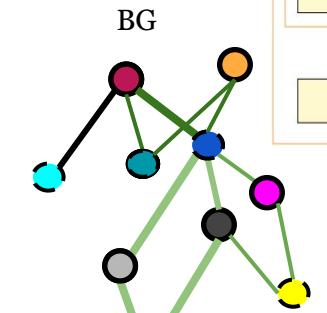
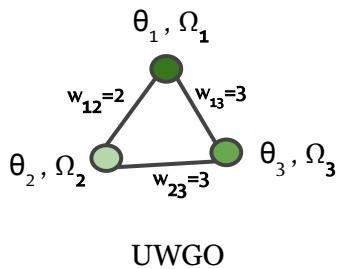
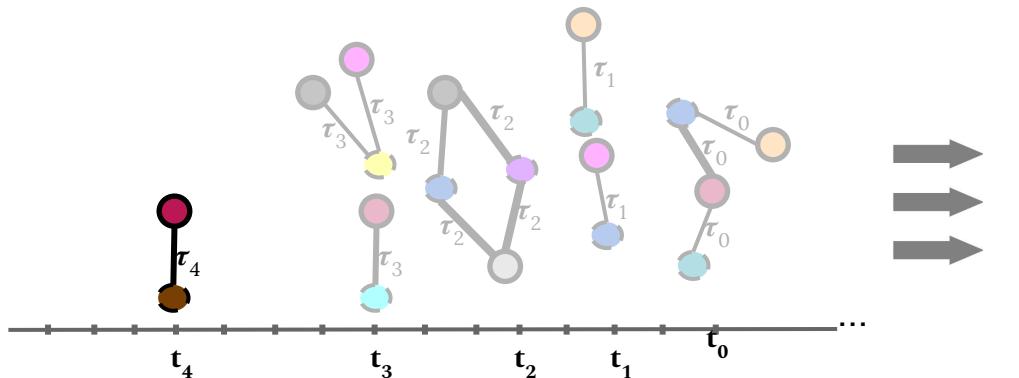
For each butterfly  $\bowtie_v$  in  $W_{BG}$   
 Create a vertex  $v$  with  $\theta_v=0, \Omega_v=0$   
 Connect  $v$  to  $n$  with  $\bowtie_n \cap \bowtie_v \neq \emptyset$   
 $W_{vn} = |N(v) \cup N(n)|$

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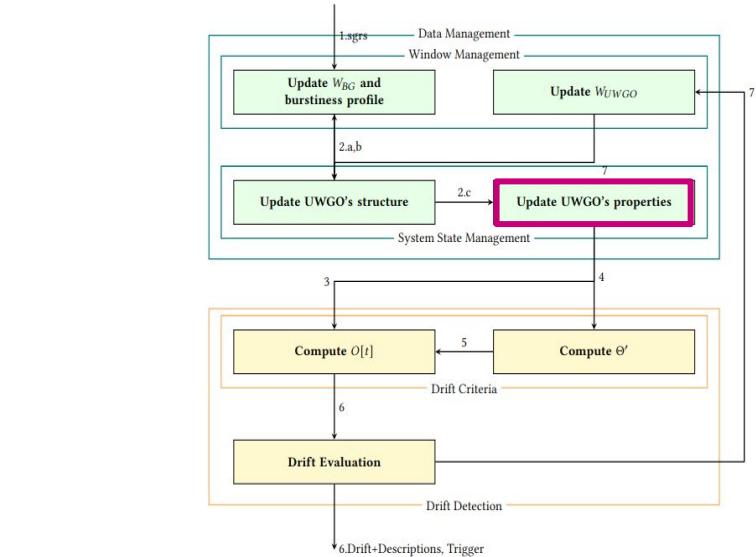
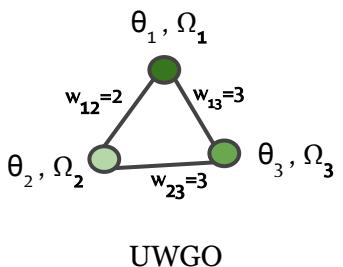
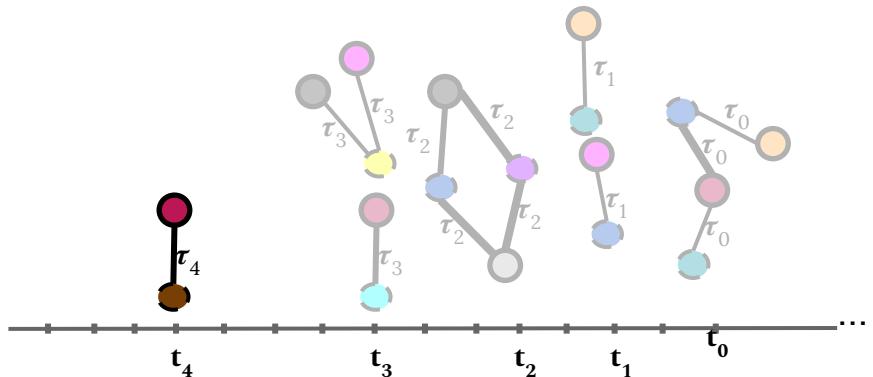
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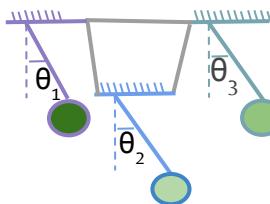


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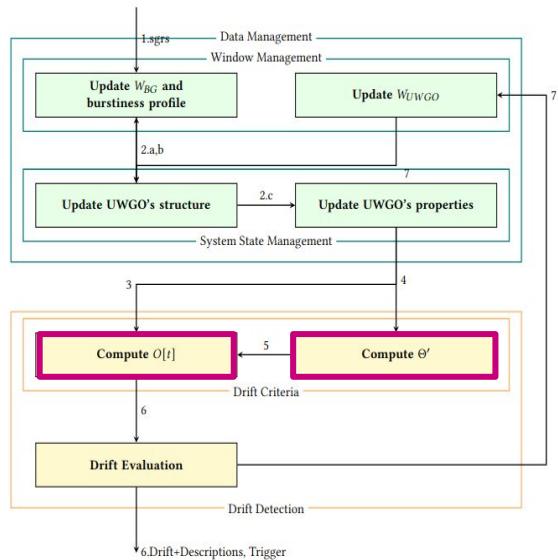
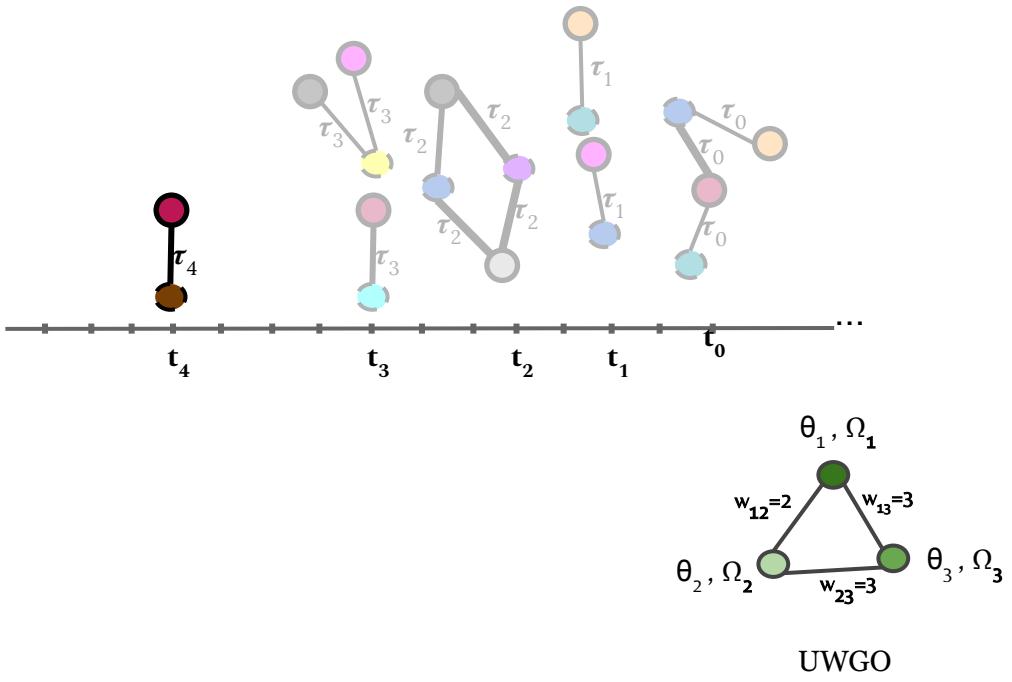
# sGradd: a framework for Streaming Graph Drift Detection



For each vertex  $v$  in  $W_{UWGO}$   
 $\theta_v := (\sum_{n \in N(v)} \text{HashCode}(n)) \% 2\pi$   
 $\Omega_v := \text{sample from a distribution}$



## sGradd: a framework for Streaming Graph Drift Detection



**Global Phase Synchronization**  $0 < O[t] < 1$

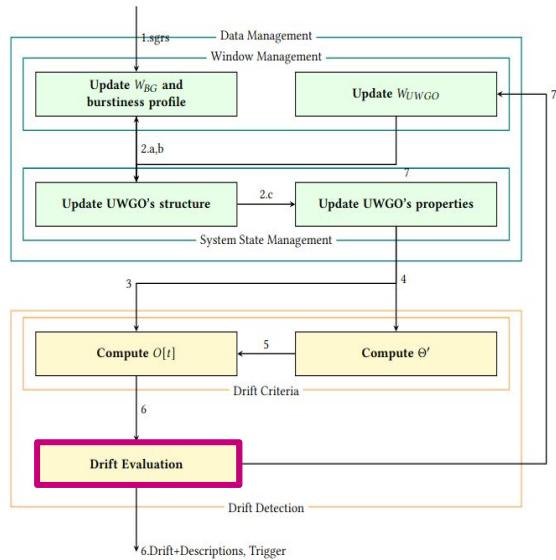
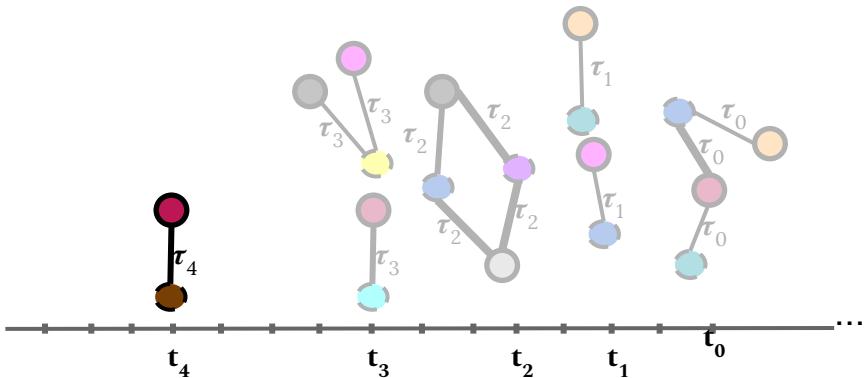
$$O_1[t_4] = \left( (\sum_{v \in V} \sin \theta_v)^2 + (\sum_{v \in V} \cos \theta_v)^2 \right)^{1/2} / |V|$$

**Kuramoto:**  $d\theta_v/dt = \Omega_v + \sum_{n \in N(v)} w_{vn} \sin(\theta_v - \theta_n)$

$\forall v \in V, \Theta_v^2 \leftarrow \text{Runge Kutta}$

$$O_2[t_4] = \left( (\sum_{v \in V} \sin \theta_v^2)^2 + (\sum_{v \in V} \cos \theta_v^2)^2 \right)^{1/2} / |V|$$

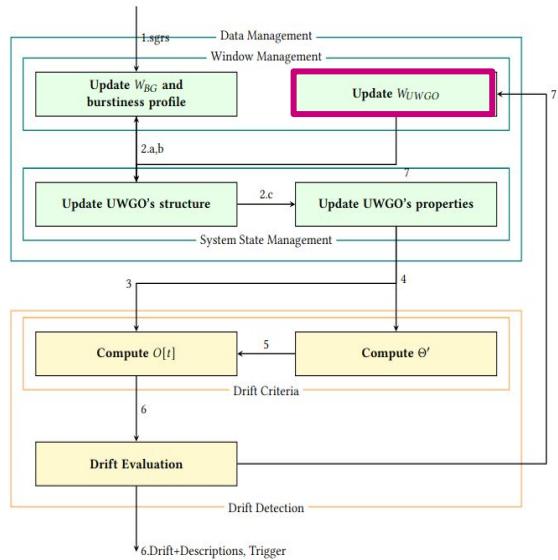
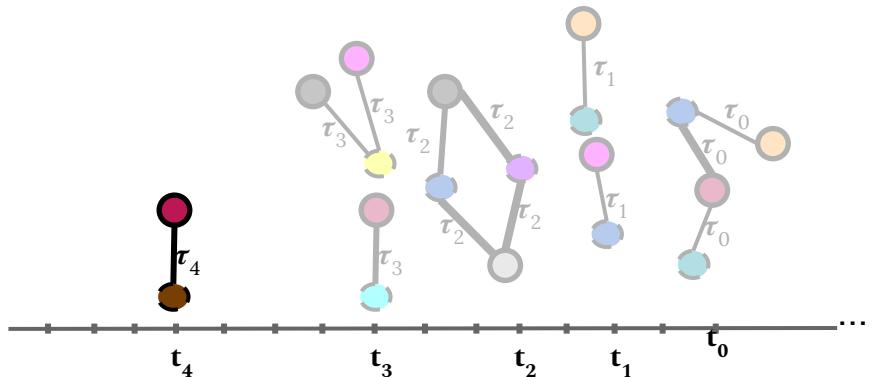
# sGradd: a framework for Streaming Graph Drift Detection



Signal a drift when

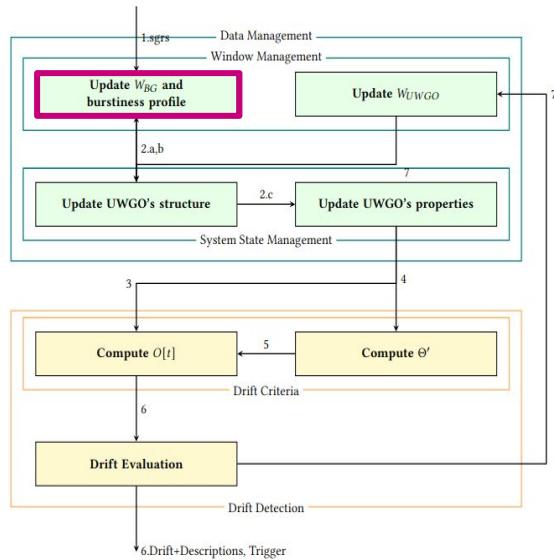
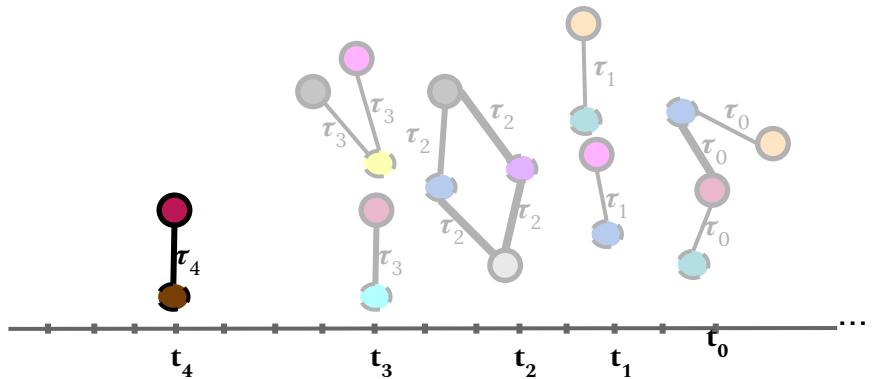
A local maximum/minimum is observed in  $O_2$ , while  $O_1$  remains steadily fixed.

# sGradd: a framework for Streaming Graph Drift Detection



If stream is bursty,  
Remove random vertices from  $W_{UWGO}$

# sGradd: a framework for Streaming Graph Drift Detection



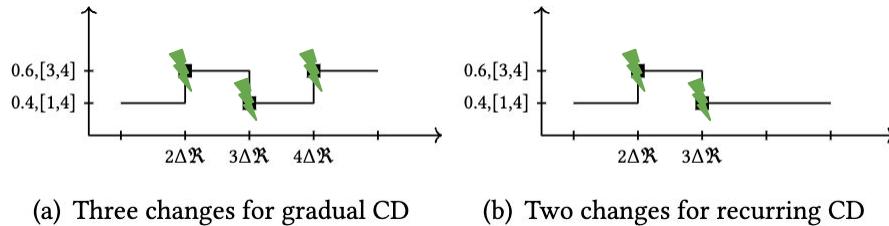
# Performance Evaluation

**Data:** 20 labelled stream with  $10^6$  records  $\leftarrow sGrow (G_0, M, \beta, \rho, [L_{min}, L_{max}]) + CD labels$

2 CD patterns

2 CD intervals:  $\Delta R = 10^5, 2 \times 10^5$

5 instances per pattern per interval



**Metrics:** True/False/Miss detection rates, Detection Delay - Average over 50 executions

**Computing Setup:** 15.6GB native memory, Intel Core i7 - 2.6GHz\*8 processor, Java

**Algorithms:** Three variants of CD detection module in sGradd

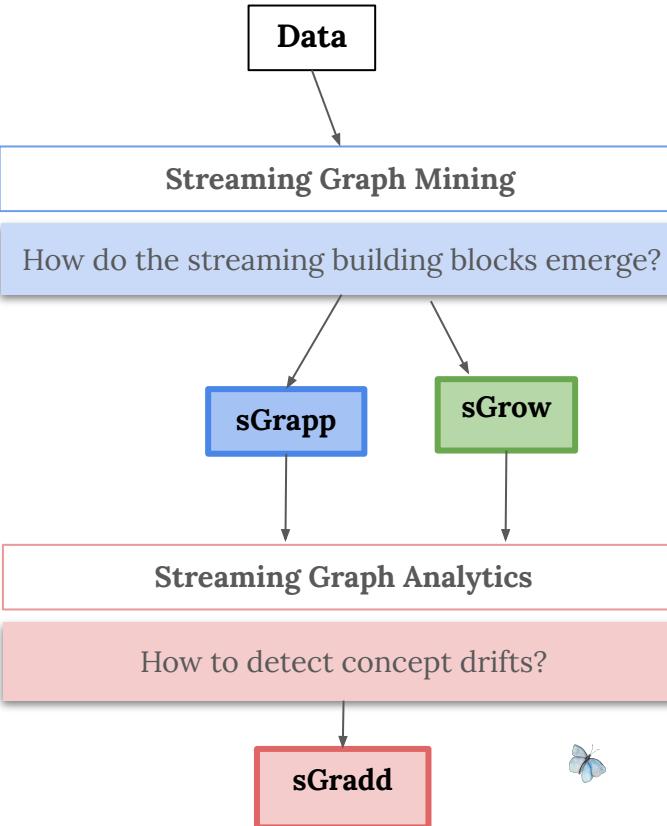
Accuracy of sGradd over streams with  $2 \times 10^5$  CD interval

Latency of sGradd over streams with  $2 \times 10^5$  CD interval

Accuracy and Latency of sGradd over streams with  $2 \times 10^5$  CD interval

Dependency of local variables and their impact on the performance

- + Improving accuracy
- + Zero false alerts
- + Average detection delay in [9, 373] & [5, 310] (s) for close and far drift intervals



A composable framework for streaming graph drift detection which

- supports any analytics (supervised or unsupervised),
- incurs low overhead, while accurately detecting the drifts,
- detects various drift types,
- explains the detected drifts' time and location,
- detects drifts without supervision,
- adapts to the streaming rate, and
- does not require input thresholds, prototype parameters, and graph attributes.

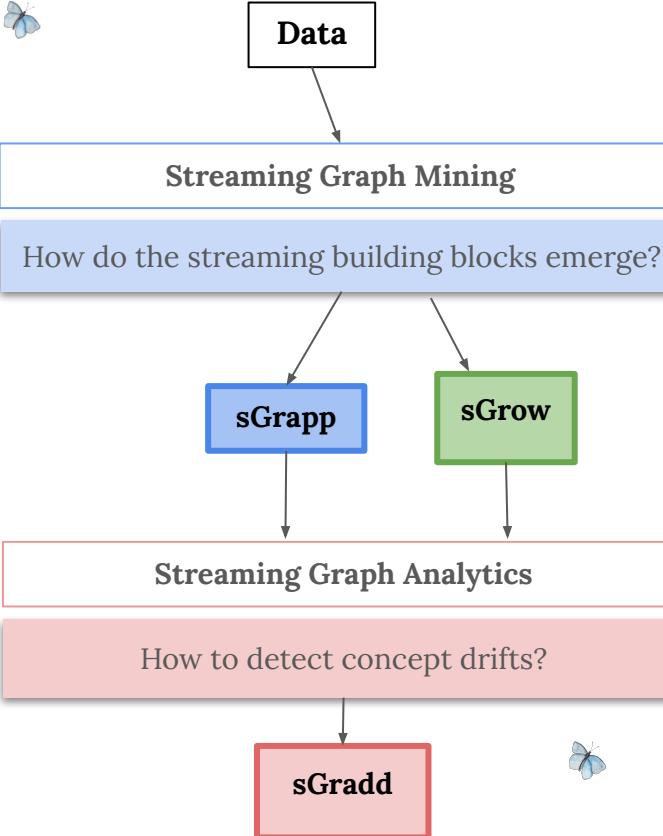
A performance evaluation which

- simulates concept drift realistically
- examines the accuracy and latency of detections effectively

# Thank you



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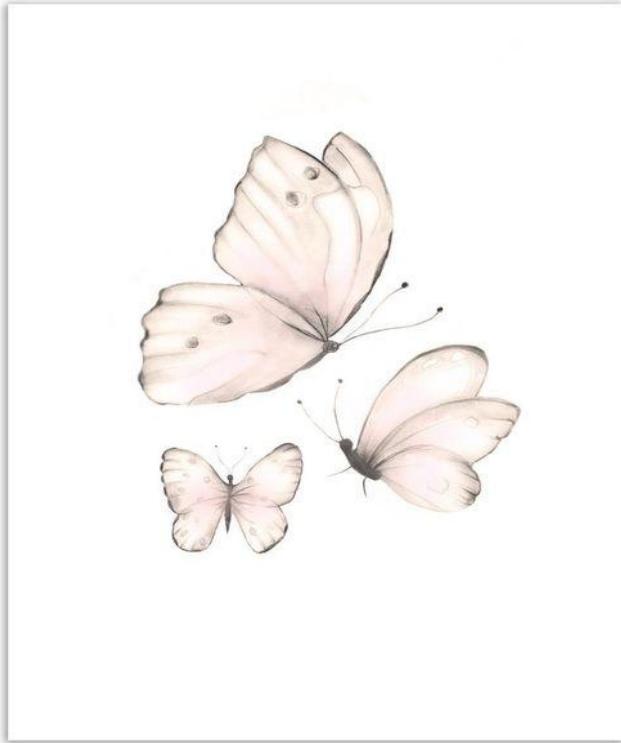


A composable framework for streaming graph drift detection which

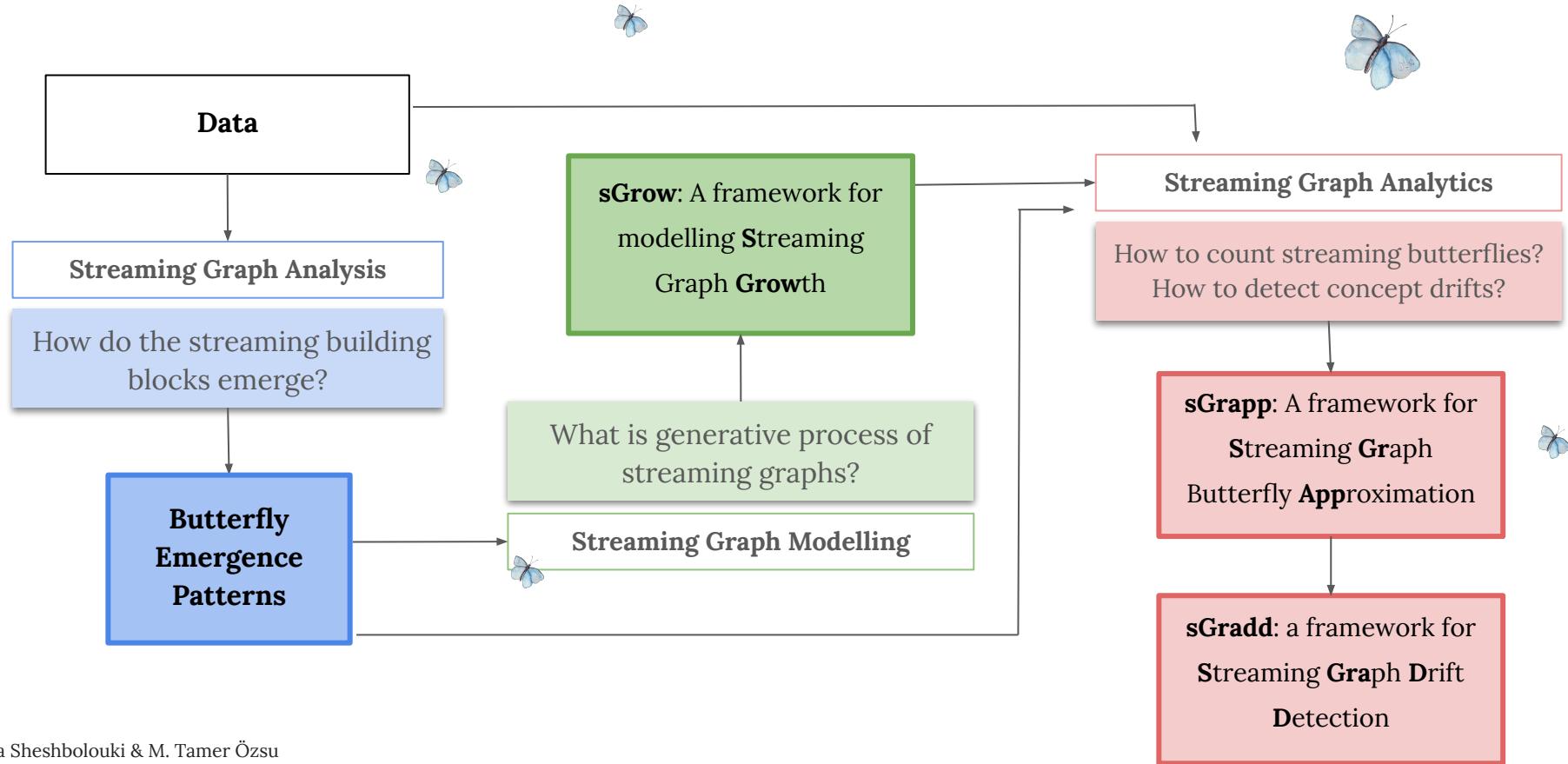
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A performance evaluation which

- simulates concept drift realistically
- examines the accuracy and latency of detections effectively



*Supporting Material*



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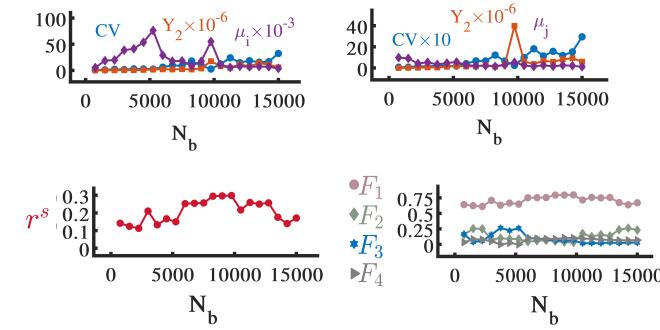
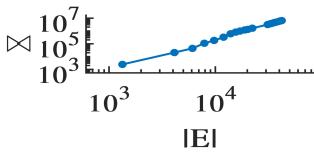
"sGrapp: Butterfly approximation in streaming graphs." ACM Transactions on Knowledge Discovery from Data 16.4 (2022): 1-43.

"sGrow: Explaining the Scale-Invariant Strength Assortativity of Streaming Butterflies." ACM Transactions on the Web (2022).



## Stream Mining

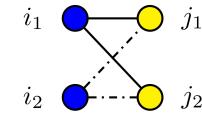
Discover and formulate the emergence patterns of butterflies in streaming graphs



## Statistical Graph Analysis

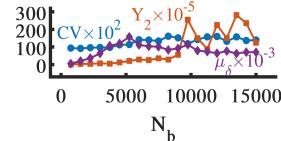
### Tools and techniques:

- A butterfly counting algorithm for static graph snapshots
- A metric for evaluating data distribution within/across streams
- Window management techniques for stream mining



### Butterfly emergence patterns:

- *Butterfly Densification Power Law*: Bursty butterfly formation and its origins
- *Scale-Invariant Strength Assortativity of Butterflies*: Butterfly mixing patterns



# Scale-Invariant Strength Assortativity of Butterflies

## Butterfly densification

The number of butterflies grows over time and at each time point it is a super-linear function of the number of edges.

## Diversification of strengths

The distribution of strengths gets broader and more diverse over time.

## Steady strength assortativity

The strength assortativity coefficient is fixed at a positive value over time due to the fixed-shaped yet growing distribution of strength-difference.

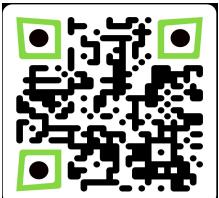
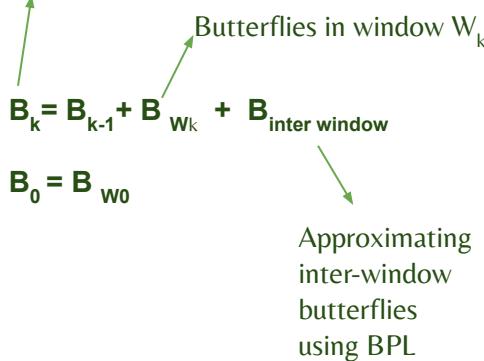
**While the new high-strength vertices with low strength-neighbors form butterflies and the variance of strength differences increases with the arrival new vertices and edges, the majority of butterflies are formed by vertices with similar strength. We refer to this phenomenon as the scale-invariant strength assortativity.**



## Graph Analytics

**sGrapp**: A framework for Streaming  
Graph Butterfly Approximation

Total butterflies until the  
end of window  $W_k$



Aida Sheshbolouki & M. Tamer Özsü

"sGrapp: Butterfly approximation in streaming graphs."

ACM Transactions on Knowledge Discovery from Data 16.4 (2022): 1-43.

## Data-driven approach for subgraph listing

### Benefits study of cohesion for

- **Marketing.** Finding groups of similar entities and recommendations
- **Information management/monitoring.** Analyzing information propagation
- **Health care.** Dynamic phenomena such as epidemic spreading

### Incremental processing powered by:

- Butterfly densification power law (BPL) for approximation
- Identifying performance bottlenecks: inter-window butterflies
- Window management adaptive to streaming rate

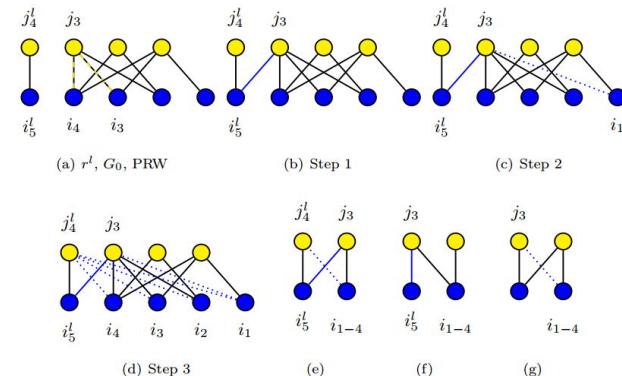
### Performance:

- Simultaneous effectiveness and efficiency
- 160x higher throughput and 0.02x lower estimation error than baselines
  - average window errors <0.05 and 0.14 in streams with uniform and non-uniform temporal distributions
  - processing throughput of  $1.5 \times 10^6$  data record per second



## Explainable Modelling

**sGrow**: A framework for modelling  
Streaming Graph **Growth**



## Micro-mechanisms for explaining the discovered patterns

### A graph traversal approach: preferential random walk

- Selecting nodes according to their weighted degree (strength preferential selection)
- Breadth-First and Depth-First traversals
- Dynamic and random number of visited nodes

### Realistic streaming record generation

- Inactivity gaps
- Timestamp assignment
- Evolving streaming rate
- Local and unbounded graph updates

### Probabilistic connections

- Random nodes
- Neighbour copying

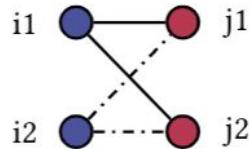
### Performance:

- Robust and efficient realization of streaming growth patterns
- Independent of initial conditions, scale and temporal characteristics, and model configurations.

Listing the butterflies in sequence of burst-based

graph snapshots of the stream:

sGrapp's core algorithm



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**Algorithm 1:** countButterflies(G)

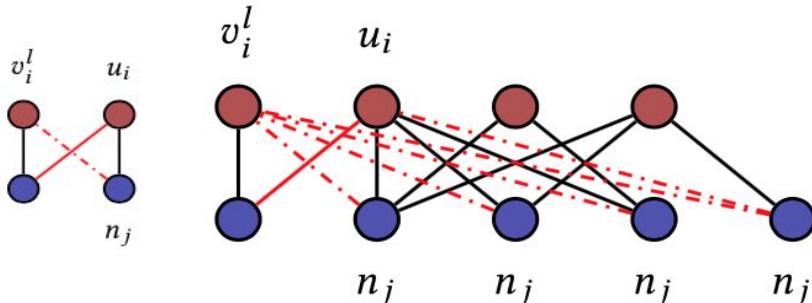
**Input:**  $G = \langle V_i \cup V_j, E_{ij} \rangle$ , static graph

**Output:**  $B_G$ , The number of butterflies in G

```
1 Butterflies  $\leftarrow \emptyset$  // An empty hashSet of quadruples
2 jNeighbors  $\leftarrow \emptyset$  // An empty Set
3 vi2s  $\leftarrow \emptyset$  // An empty Set
4 /* loop over  $v_{i1} \in V_i$  if  $K_i < K_j$ , otherwise loop over  $v_{j1} \in V_j$  */
5 for  $v_{i1} \in V_i$  do
6   jNeighbors  $\leftarrow N_{v_{i1}}$  // j-neighbors of vertex  $v_{i1}$ 
7   for  $index1 \in [1, size(jNeighbors)]$  do
8      $v_{j1} \leftarrow jNeighbors[index1]$ 
9     for  $index2 \in [index1 + 1, size(jNeighbors)]$  do
10        $v_{j2} \leftarrow jNeighbors[index2]$ 
11       vi2s  $\leftarrow N_{v_{j1}} \cap N_{v_{j2}}$  // common i-neighbors
12       Butterflies.add( $[v_{i1}, v_{j1}, v_{i2}, v_{j2}]$ )
13
14 12  $B_G \leftarrow size(Butterflies)$ 
```

---

# Explaining the Scale-invariant strength assortativity of butterflies: sGrow's algorithms



Algorithm 4: Add Burst

```

1 Function addBurst( $v_i^l, v_j^l, PRW_i, G, \mathfrak{R}, \rho$ )
2   for each  $u_i \in PRW_i$  do
3     |Ni(ui)|-caterpillars in butterflies
4       Add a new sgr  $\langle u_i, v_j^l, \omega', v_j^l, \tau \rangle$  to  $\mathfrak{R}$  and  $G$ 
5       if  $\text{coin}(\rho)$  is Head then
6          $z_j \leftarrow$  Select a random j-vertex
7          $\omega' \leftarrow$  a random integer in  $[1, 5]$ 
8         Add a new sgr  $\langle u_i, z_j, \omega', \text{Min}(u_i.\tau, z_j.\tau) \rangle$  to  $\mathfrak{R}$  and  $G$ 
9       for each  $n_j \in N_j(u_i)$  do                                // in bursty streams with  $b > 2$ 
10      if  $\text{coin}(\rho)$  is Head then
11         $\omega' \leftarrow$  a random integer in  $[1, 5]$ 
12        Add a new sgr  $\langle v_i^l, n_j, \omega', n_j, \tau \rangle$  to  $\mathfrak{R}$  and  $G$ 

```

Algorithm 1: Graph Model

```

Input:  $\rho$ : connection probability,  $M$ : number of isolated edges,  $\beta$ : slide parameter,
 $[L_{min}, L_{max}]$ : PRW's length range
Output:  $\mathfrak{R}$ , sequence of streaming graph records
1  $G \leftarrow G_0 = (V_0, E_0)$                                          // computational graph
2  $\mathfrak{R} \leftarrow E_0$                                                  // sequence of sgrs
3  $\tau \leftarrow 1$  last timestamp in  $G_0$                                 // timestamp
4  $t \leftarrow 0$                                                        // timestep
5  $W^b \leftarrow$  first timestamp in  $G_0$                                 // sliding window's beginning border
6 while true do
7    $t \leftarrow t + 1$ 
8   Add  $m \in [0, M]$  new sgrs  $r^{l=0, \dots, m}$  to  $\mathfrak{R}$  and  $G$ 
9   for each  $r^{l=0, \dots, m} = \langle v_i^l, v_j^l, \omega_{ij}^l, \tau \rangle$  do
10     $\omega \leftarrow$  a random integer in  $[-1, 5]$ 
11    switch  $\omega$  do
12      case -1 do
13        Remove any edge between  $v_i^l$  and  $v_j^l$  from  $\mathfrak{R}$  and  $G$ .
14      case 0 do
15        No operation
16      otherwise do
17         $init_j \leftarrow SPS(V_j)$ 
18         $L \leftarrow$  a random integer in  $[L_{min}, L_{max}]$ 
19         $(PRW_i, PRW_j) \leftarrow PRW(init_j, \text{false}, G, L)$ 
20        addBurst( $v_i^l, v_j^l, PRW_i, G, \mathfrak{R}, \rho$ )
21        addBurst( $v_i^l, v_j^l, PRW_j, G, \mathfrak{R}, \rho$ )
22         $\tau \leftarrow \tau + \frac{(\omega'-5)(\omega'-4)(\omega'-3)}{2}$ 
23   Remove any newly added vertex  $v_i^l$  and  $v_j^l$  with less than 2 neighbors from  $G$ 
24    $\tau \leftarrow \tau + 1$ 
25    $W^b \leftarrow W^b + \beta$ 
26   if  $t = \beta$  then
27     Remove any edge with timestamp less than  $W^b$  from  $G$ 
28    $t \leftarrow 0$ 

```